

Physics 1A Discussion (Week 2)

Problem 1 (problem 1.53 on page 30 of the textbook)

BIO Estimate the number of atoms in your body. (Hint: Based on what you know about biology and chemistry, what are the most common types of atoms in your body? What is the mass of each type of atom?)

The leading mass fraction of each atom species in your body is,

3% Nitrogen	with atomic mass: $14.0u$
11% Hydrogen	with atomic mass: $1.0u$
19% Carbon	with atomic mass: $12.0u$
67% Oxygen	with atomic mass: $16.0u$

We also know that the total number of particles is simply the sum of the number of particles over the different types of particles.

$$N_{\text{Total}} = N_{\text{N}} + N_{\text{H}} + N_{\text{C}} + N_{\text{O}} \quad (1)$$

Let m be the atomic mass of a particular atom. The collective mass M of the particles with that atomic mass is given by,

$$M = mN \quad (2)$$

Dividing over m we have,

$$\frac{M}{m} = N \quad (3)$$

We can use this to find the number of particles for all the different atoms. Starting with N_{N} we know that this corresponds to 3% of the total mass so it follows that

$$N_{\text{N}} = \frac{0.03 \times 60 \text{ kg}}{14u} \quad (4)$$

Using the unit conversion $1.0u = 1.66 \times 10^{-27} \text{ kg}$.

$$N_{\text{N}} = \frac{0.03 \times 60 \text{ kg}}{14.0 \times (1.66 \times 10^{-27} \text{ kg})} \sim 7.7 \times 10^{25} \quad (5)$$

Similarly for all the other N 's we get,

$$N_{\text{H}} = \frac{0.11 \times 60 \text{ kg}}{1.0 \times (1.66 \times 10^{-27} \text{ kg})} \sim 3.98 \times 10^{27} \quad (6)$$

$$N_{\text{C}} = \frac{0.19 \times 60 \text{ kg}}{12.0 \times (1.66 \times 10^{-27} \text{ kg})} \sim 5.7 \times 10^{26} \quad (7)$$

$$N_{\text{O}} = \frac{0.67 \times 60 \text{ kg}}{16.0 \times (1.66 \times 10^{-27} \text{ kg})} \sim 1.5 \times 10^{27} \quad (8)$$

Plugging into the formula for total N we get,

$$N_{\text{Total}} = N_{\text{N}} + N_{\text{H}} + N_{\text{C}} + N_{\text{O}} \approx 6 \times 10^{27}. \quad (9)$$

Therefore the human body is composed of approximately 6×10^{27} atoms.

Problem 2(problem 1.72 on page 30 of the textbook)

Ricardo and Jane are standing under a tree in the middle of a pasture. An argument ensues, and they walk away in different directions. Ricardo walks 26.0 m in direction 60.0° west of north. Jane walks 16.0 m in a direction 30° south of west. They then stop and turn to face each other.

(a) What is the distance between them?

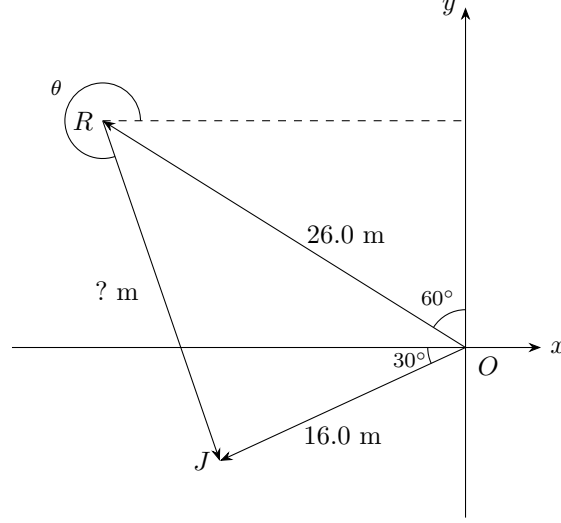


Figure 1: The positions of Ricardo and Jane in the xy -plane. The position of the tree is set to be the origin, and an angle θ corresponds to the angle of $\vec{RJ}$ measured from the $+x$ axis toward the $+y$ axis that appears in the problem (b).

The position of Ricardo(R) and Jane(J) are given by

$$\vec{OR} = \begin{pmatrix} 26.0 \cos(60^\circ + 90^\circ) \text{ m} \\ 26.0 \sin(60^\circ + 90^\circ) \text{ m} \end{pmatrix} = \begin{pmatrix} -13.0\sqrt{3} \text{ m} \\ 13.0 \text{ m} \end{pmatrix}, \quad \vec{OJ} = \begin{pmatrix} 16.0 \cos(30^\circ + 180^\circ) \text{ m} \\ 16.0 \sin(30^\circ + 180^\circ) \text{ m} \end{pmatrix} = \begin{pmatrix} -8.0\sqrt{3} \text{ m} \\ -8.0 \text{ m} \end{pmatrix}. \quad (10)$$

Because $\vec{OJ} = \vec{OR} + \vec{RJ} \Rightarrow \vec{RJ} = \vec{OJ} - \vec{OR}$, we get

$$\vec{RJ} = \begin{pmatrix} -8.0\sqrt{3} \text{ m} - (-13.0\sqrt{3} \text{ m}) \\ -8.0 \text{ m} - 13.0 \end{pmatrix} = \begin{pmatrix} 5.0\sqrt{3} \text{ m} \\ -21.0 \text{ m} \end{pmatrix}. \quad (11)$$

Then the magnitude of $\vec{RJ}$ is given by

$$|\vec{RJ}| = \sqrt{(5.0\sqrt{3})^2 + (21.0)^2} \text{ m} = 22.7 \text{ m} \quad (12)$$

This is the distance between them.

(b) In what direction should Ricardo walk to go directly toward Jane?

By using the direction angle θ of $\vec{RJ}$ measured from the $+x$ axis toward the $+y$ axis, you can express $\vec{RJ}$ by

$$\vec{RJ} = \begin{pmatrix} |\vec{RJ}| \cos \theta \\ |\vec{RJ}| \sin \theta \end{pmatrix}. \quad (13)$$

Since we already have the component display of $\vec{RJ}$ and the magnitude $|\vec{RJ}|$, we see that the angle θ satisfies

$$\begin{aligned} \cos \theta &= \frac{5.0\sqrt{3} \text{ m}}{22.7 \text{ m}} \sim 0.38, \\ \sin \theta &= \frac{-21.0 \text{ m}}{22.7 \text{ m}} \sim -0.93 \\ \Rightarrow \theta &\sim -68^\circ \end{aligned} \quad (14)$$

Then R should walk 22.7 m in a direction 68° south of the west.

Problem 3 (problem 1.82 on page 31 of the textbook)

Vector $\vec{A}$ has magnitude 5.00 m and lies in the xy -plane in a direction 53.0° from the $+x$ -axis measured toward the $+y$ -axis. Vector $\vec{B}$ has magnitude 8.00 m and a direction you can adjust.

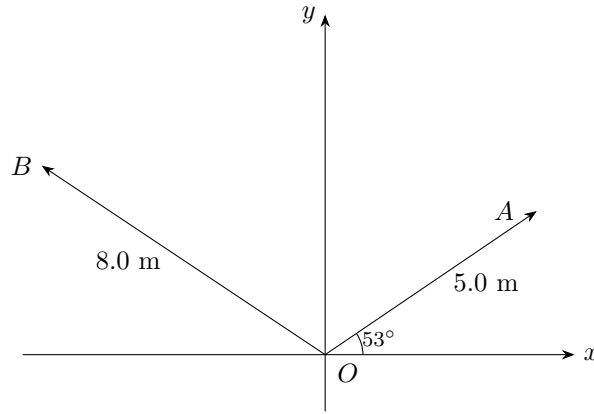


Figure 2: Illustration of $\vec{A}, \vec{B}$ in the xy -plane, where the direction of $\vec{B}$ is set randomly because it can be adjusted later by varying the directional angle ϕ , or the opening angle $\theta = \angle AOB$

(a) You want the vector product $\vec{A} \times \vec{B}$ to have a positive z -component of the largest possible magnitude what direction should you select for vector $\vec{B}$?

First let's state the magnitude of the cross product

$$|\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \sin(\theta). \quad (15)$$

The sine function is maximized at 90° , and $\vec{A}$ is already oriented 53° away from the x -axis. $\vec{B}$ must have an angle ϕ ,

$$\phi = 53^\circ + 90^\circ = 143^\circ \quad (16)$$

(b) What is the direction of $\vec{B}$ for which $\vec{A} \times \vec{B}$ has the most negative z -component?

The sine function is minimized at 270° or -90° and $\vec{A}$ is already oriented 53° away from the x -axis. $\vec{B}$ must have an angle ϕ ,

$$\phi = 53^\circ - 90^\circ = -37^\circ \quad (17)$$

(c) What are the two directions of $\vec{B}$ for which $\vec{A} \times \vec{B} = \vec{0}$?

The sine function is zero at 0° or 180° and $\vec{A}$ is already oriented 53° away from the x -axis. $\vec{B}$ must have an angle ϕ ,

$$\phi = 53^\circ, \text{ and } \phi = 53^\circ + 180^\circ = 233^\circ \quad (18)$$